

Mark Scheme (Results)

Summer 2013

International GCSE Further Pure Mathematics  
Paper 2 (4PM0/02)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC – special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

If no working is shown then correct answers may score full marks.

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can still gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

- **Follow through marks**

Follow through marks which involve a single stage of calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignore subsequent working**

It is appropriate to ignore subsequent working when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent working when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

## General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

### Method mark for solving 3 term quadratic:

1. Factorisation  $(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x =$   
 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x =$

2. Formula

Attempt to use correct formula (with values for  $a$ ,  $b$  and  $c$ ).

3. Completing the square

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ ,  $q \neq 0$ , leading to  $x = \dots$

### Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

### Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

### Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

### Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

### Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1.	<p>(a) Area <math>ABC = \frac{1}{2} \times 10 \times 16 \times \sin 35 = 45.9 \text{ cm}^2</math></p> <p>(b) <math>BC^2 = 10^2 + 16^2 - 2 \times 10 \times 16 \cos 35</math>  <math>BC = \sqrt{93.87} = 9.69</math>  <math>\cos B = \frac{10^2 + 93.87 - 16^2}{2 \times 10 \times \sqrt{93.87}} \quad (= -0.3206)</math>  <math>\angle B = 108.7^\circ</math>  <i>alternative</i> (last three marks of part (b))  <math>\frac{16}{\sin B} = \frac{9.69}{\sin 35} \Rightarrow \sin B = \frac{16 \times \sin 35}{9.69} = 0.9472</math>      M1 A1ft  <math>\Rightarrow \angle B = 71.3^\circ</math> or <math>\angle B = 108.7^\circ</math>  so  <math>\angle A = 35^\circ, \angle B = 71.3^\circ, \angle C = 73.7^\circ</math> or <math>\angle A = 35^\circ, \angle B = 108.7^\circ, \angle C = 36.3^\circ</math>  since <math>AC</math> is the longest side, <math>\angle B</math> is the largest angle, so <math>\angle B = 108.7^\circ</math>      A1  <i>alternative</i> (last three marks of part (b))  <math>\frac{10}{\sin C} = \frac{9.69}{\sin 35^\circ} \Rightarrow \sin C = \frac{10 \sin 35^\circ}{9.69} = 0.5920</math>      M1 A1ft  <math>\angle C = 36.3^\circ</math> (must be acute)  <math>\angle B = 180^\circ - 35^\circ - 36.3^\circ = 108.7^\circ</math>      A1</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>A1 (7)</p>

### Notes for Question 1

(a)

M1 for any *complete* method for obtaining the area of  $\triangle ABC$ . Mark scheme uses  $\frac{1}{2}ab \sin C$

formula, but the perpendicular height from  $B$  can be found (not nec

correct value) and  $\frac{1}{2} \times \text{base} \times \text{height}$  used.

A1cao for 45.9 (cm<sup>2</sup>) **must be 3 sf**

(b)

M1 for the cosine rule, in either form, with  $BC$  as the unknown

A1 for making  $BC$  or  $BC^2$  the subject and all numbers correct.

M1 for using the cosine rule to obtain a numerical expression for value of  $\cos B$  or or sine rule (either way up) with  $\sin B$  as the unknown

A1ft for correct numbers in the sine or cosine rule, follow through their value for  $BC^2$  (or  $BC$ ).

$\sin B$  or  $\sin C$  must now be the subject.

A1cao for 108.7°. Allow with more digits if rounding penalised in (a).

*Alternative for (b):* Uses the perp height. Marks for (a) cannot be awarded here, so no mark for finding the height in (b).

Perp from $B = BX = 10 \sin 35$	
$AX = 10 \cos 35 \Rightarrow XC = 16 - 10 \cos 35$	M1A1
$\tan XBC = \frac{XC}{BX} = \frac{16 - 10 \cos 35}{10 \sin 35}, \quad \angle XBC =$	M1,A1
Reqd angle is $55 + 53.7 = 103.7$	A1





Question Number	Scheme	Marks
3.	<p>(a) (i) <math>\int (1+3x-\frac{2}{x^2})dx = x + 3\frac{x^2}{2} - 2\frac{x^{-1}}{-1} [+c]</math></p> <p>(ii) <math>\int_1^2 (1+3x-\frac{2}{x^2})dx = \left[ x + 3\frac{x^2}{2} - 2\frac{x^{-1}}{-1} \right]_1^2</math>  <math>= (2+6+1) - (1+\frac{3}{2}+2) = 9 - 4\frac{1}{2} = 4\frac{1}{2} \quad *</math></p> <p>(b) (i) <math>\int 3\sin 2x dx = \frac{-3\cos 2x}{2} [+c]</math></p> <p>(ii) <math>\int_0^{\frac{\pi}{6}} 3\sin 2x dx = \left[ \frac{-3\cos 2x}{2} \right]_0^{\frac{\pi}{6}}</math>  <math>= -\frac{3}{2}(\cos \frac{\pi}{3} - \cos 0)</math>  <math>= -\frac{3}{2}(\frac{1}{2} - 1) = \frac{3}{4} \quad *</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (8)</p>

### Notes for Question 3

(a)

(i) M1 for attempting the integration, (at least one term must show an increase of power - see start of doc)

A1 for  $x + 3\frac{x^2}{2} - 2\frac{x^{-1}}{-1} (+c)$  No simplification needed, constant not needed

(ii) M1 for substituting the given limits in their result for (i). This is a "show" question and also states "hence", so the substitution **must** be seen.

A1cso for  $4\frac{1}{2}$  \* (Given answer, unlikely to be a correct solution if (i) not fully correct, but

candidate may have integrated again and correctly this time (marks for (i) can be given as all in (a)). Also, check substitution correct.)

(b)

(i) M1 for attempting to integrate.  $\cos 2x$  must be seen. Minus sign not needed. The  $\frac{1}{2}$  may be omitted, but if  $2\cos 2x$  is seen assume differentiation and award M0. (Should be

$\pm 3\cos 2x$  or  $\pm \frac{3}{2}\cos 2x$ )

A1 for  $\frac{-3\cos 2x}{2} (+c)$  constant not needed

*Alternative:* (Not likely to be seen often)

M1 change to  $\int 3 \times 2 \sin x \cos x dx$  and attempt to integrate. Must be attempting to integrate the **correct** function and  $\pm \sin^2 x$  or  $\pm \cos^2 x$  must be seen. Accept 3 or 6 but not 12

A1 for  $3\sin^2 x$  or  $-3\cos^2 x (+c)$

(ii) M1 for substituting the given limits (sub must be seen) in their result for (i). Can be in terms of cosines (or sines in alternative) or may go straight to the corresponding numbers (but not the final answer) Accept 0 for  $\cos 0$ .

A1cso for  $\frac{3}{4}$  \* Not nec to see  $-\frac{3}{4} + \frac{3}{2}$  for this mark.

As in (a), a candidate can re-start in (ii) and gain the marks for (i) if they have not already been awarded.

Question Number	Scheme	Marks
4.	<p>(a) <math>t_n = 1 \times r^{n-1} = r^{n-1}</math></p> <p>(b) <math>r^{n-1} + r^n = r^{n+1}</math>  <math>\Rightarrow 1 + r = r^2</math>  <math>\Rightarrow r^2 - r - 1 = 0</math>  <math>\Rightarrow r = \frac{1 \pm \sqrt{1+4}}{2}</math>  <math>\Rightarrow r = \frac{1 + \sqrt{5}}{2}</math> since <math>r &gt; 0</math></p> <p>(c) <math>t_1 = 1, t_2 = r = \frac{1 + \sqrt{5}}{2}</math>  <math>t_3 = t_1 + t_2 = 1 + \frac{1 + \sqrt{5}}{2} \left( = \frac{3 + \sqrt{5}}{2} \right)</math>  <math>t_4 = t_2 + t_3 = \frac{1 + \sqrt{5}}{2} + \frac{3 + \sqrt{5}}{2} = \frac{4 + 2\sqrt{5}}{2} = 2 + \sqrt{5}</math>  <i>alternative</i>  (c) <math>t_4 = \left( \frac{1 + \sqrt{5}}{2} \right)^3 = \frac{1 + 3\sqrt{5} + 3(\sqrt{5})^2 + (\sqrt{5})^3}{8}</math>  <math>= \frac{1 + 3\sqrt{5} + 15 + 5\sqrt{5}}{8}</math> oe  <math>= \frac{16 + 8\sqrt{5}}{8} = 2 + \sqrt{5}</math></p>	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>A1 (8)</p> <p>M1</p> <p>A1</p> <p>A1</p>

#### Notes for Question 4

(a) B1 for  $t_n = 1 \times r^{n-1}$  (or  $t_n = r^{n-1}$ )

(b)

M1 for forming an equation  $r^{n-1} + r^n = r^{n+1}$  follow through their expression for  $t_n$  (ie their expression from (a) in  $t_n + t_{n+1} = t_{n+2}$ )

A1 for dividing by  $r^{n-1}$  to obtain a correct 3 term quadratic - terms in any order - from a **correct** initial equation (ie eg  $t_n = r^n$  in (a) giving the equation  $r^n + r^{n+1} = r^{n+2}$  would score B0M1A0)

*Alternative:* Make  $r = 1$  so  $t_1 + t_2 = t_3$  M1 and so  $1 + r = r^2$  A1

M1 for solving **their** 3 term quadratic equation - see start of doc for information about solving quadratic equations. Either the general formula must be quoted correctly or the full substitution seen (as answer

given) **OR** subst.  $r = \frac{1 + \sqrt{5}}{2}$  (M1) then if everything correct including a conclusion, give A1

A1cso for  $r = \frac{1 + \sqrt{5}}{2}$  \* as  $r > 0$ . Since this is a given answer **we must see**  $r > 0$  somewhere. A correct

general formula with  $\pm$  needed or both solutions shown and the positive chosen with the reason. If

$r = \frac{1 + \sqrt{1+4}}{2} \Rightarrow r = \frac{1 + \sqrt{5}}{2}$   $r > 0$  is given, award M1A0.

### Notes for Question 4 Continued

(c)

M1 for obtaining  $t_3$  by using  $t_3 = t_1 + t_2$  their numerical  $t_1$  and  $t_2$

A1 for  $t_3 = 1 + \frac{1 + \sqrt{5}}{2}$  oe

A1cao and cso for  $(t_4) = 2 + \sqrt{5}$

*Alternatives for (c)*

M1 for using **their** formula found in (a) and attempting the expansion

A1 for  $\frac{1 + 3\sqrt{5} + 15 + 5\sqrt{5}}{8}$  oe      A1 for  $2 + \sqrt{5}$

$t_4 = t_3 + t_2 = t_2 + t_1 + t_2$ $= 2 \times \frac{(1 + \sqrt{5})}{2} + 1$ $= 2 + \sqrt{5}$	<p>M1A1</p> <p>A1</p>
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By calculator:  $\left(\frac{1 + \sqrt{5}}{2}\right)^3 = 2 + \sqrt{5}$  scores M1A1A1, but an incorrect (or partially correct) answer scores

M0A0A0

Question Number	Scheme	Marks
5.	<p>(a) <math>30 = 2y + 2x + \pi x</math>  <math>2y = 30 - 2x - \pi x</math>  <math>y = 15 - x - \frac{1}{2}\pi x</math> oe</p> <p>(b) <math>A = 2xy + \frac{1}{2}\pi x^2</math>  <math>= x(30 - 2x - \pi x) + \frac{1}{2}\pi x^2</math>  <math>= 30x - 2x^2 - \frac{1}{2}\pi x^2</math> *</p> <p>(c) <math>\frac{dA}{dx} = 30 - 4x - \pi x</math>  At maximum, <math>\frac{dA}{dx} = 0 \Rightarrow 30 - 4x - \pi x = 0</math>  <math>\Rightarrow x = \frac{30}{4 + \pi}</math> [= 4.201]  <math>\frac{d^2A}{dx^2} = -4 - \pi &lt; 0 \Rightarrow</math> maximum</p> <p>Maximum area = <math>30\left(\frac{30}{4 + \pi}\right) - 2\left(\frac{30}{4 + \pi}\right)^2 - \frac{1}{2}\pi\left(\frac{30}{4 + \pi}\right)^2 = 63</math> to 2SF</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 (11)</p>

### Notes for Question 5

(a) M1 for setting up the equation  $30 = 2y + 2x + \pi x$  ... Allow with a circle (ie  $2\pi x$ ) and the missing side ( $4x$  instead of  $2x$ )

A1cao for re-arranging to get  $y = 15 - x - \frac{1}{2}\pi x$  oe

(b) M1 for  $A = 2xy + \frac{1}{2}\pi x^2$  and sub **their** expression for  $y$  (semicircle or circle  $\pi x^2$  for M1)

A1cso for  $A = 30x - 2x^2 - \frac{1}{2}\pi x^2$  \*

**NB: Watch for double sign errors in (a) and (b) and deduct A marks as appropriate.**

(c)

M1 for differentiating the **given** expression for  $A$  wrt  $x$

M1 for equating **their** differential to 0 (NB this is not a dependent M mark)

A1 for obtaining  $x = \frac{30}{4 + \pi}$

M1 for attempting the second differential

A1 for stating this differential is negative, so a maximum, **providing** the second differential is correct. A **correct** differential with no mention of  $\frac{d^2A}{dx^2}$  can get M1A1

*Alternatives for the last two marks:*

M1 for testing the sign of  $\frac{dA}{dx}$  on either side of **their**  $x$ . Numerical calculations must be seen. Only one turning point so values chosen need not be close to their  $x$ .

A1 for correct results and the conclusion, **providing**  $\frac{dA}{dx}$  is correct

OR: M1 Graph of  $A = 30x - 2x^2 - \frac{1}{2}\pi x^2$  is a parabola/quadratic opening downwards (must be stated or a sketch shown)

A1 turning point is therefore a maximum.

**Then:**

M1 for substituting **their positive**  $x$  in the **given** expression for  $A$ .

A1cao for  $A_{\max} = 63$  (cm<sup>2</sup>) **Must be 2 sf.**

Question Number	Scheme	Marks
6.	<p>(a) <math>p(-2) = 2(-2)^3 + 13(-2)^2 - 17(-2) - 70</math> or <math>p(x) = (x+2)(2x^2+9x-35)</math>            So <math>(x+2)</math> is a factor of <math>p(x)</math>,            Hence, by the factor theorem,  <math display="block">= -16 + 52 + 34 - 70 = 0</math>  <math>p(-2) = 0^*</math></p> <p>(b) <math>(x+2)(2x^2+9x-35) = 0</math>  <math>(x+2)(2x-5)(x+7) = 0</math>  <math>x = -2, x = 2\frac{1}{2}, x = -7</math></p>	<p>M1 A1  M1 A1 M1dep A1 (6)</p>

### Notes for Question 6

(a)  
 M1 for substituting  $x = -2$  in the given function  
 A1cso for  $p(-2) = 0$  \* ( $-16 + 52 + 34 - 70$  must be seen)

(b)  
 M1 for writing  $p(x)$  as  $(x+2) \times$  a quadratic factor with at least 2 terms (by inspection) or dividing to obtain the quadratic factor (not necessarily correct). The quadratic must start with  $2x^2$ .  
 A1 for  $(x+2)(2x^2+9x-35)$  or just  $2x^2+9x-35$   
 M1dep for solving the quadratic (usual rules)  
 A1 for the **three** solutions,  $x = -2, 2\frac{1}{2}, -7$  Often  $-2$  is missing.  
 NB: Do not penalise if " $= 0$ " not seen.

*Alternative:*

(b)  
 M1 for using the factor theorem again with any one of  $x = \pm 5, \pm 7, \pm \frac{5}{2}, \pm \frac{7}{2}$   
 A1 for a correct result for  $x = \frac{5}{2}$  or  $-7$   
 M1 finding a third value, any valid method  
 A1 for the **three** solutions,  $x = -2, 2\frac{1}{2}, -7$  Often  $-2$  is missing.  
 NB: No working shown (ie calculator solution):  
 If all **three** solutions shown and correct 4/4  
 If one (or more) missing or incorrect 0/4



Question Number	Scheme	Marks
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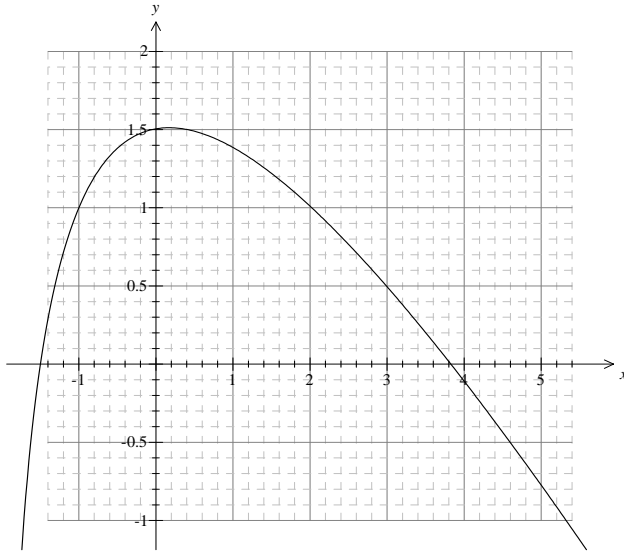
7.

(a)

$x$	-1	0	1	2	3	4	5
$y$	1	1.51	1.39	<b>1.01</b>	<b>0.49</b>	<b>-0.11</b>	-0.77

B1 B1

(b)



B1ft plot  
B1ft curve

(c)  $5 \log(x+2) - x = \frac{3}{4}$

M1

Reading from graph where  $y = \frac{3}{4}$  gives  $x = 2.5$  to 1dp

A1

(d)  $x+2 = 10^{\frac{1}{2}x}$

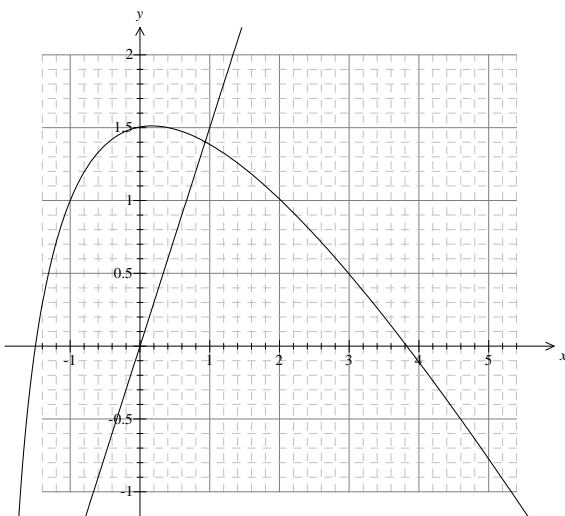
$\log(x+2) = \frac{1}{2}x$

$5 \log(x+2) - x = 1\frac{1}{2}x$

M1 A1

from graph,  $x = 0.9$  to 1dp

M1dep A1  
(10)



### Notes for Question 7

*Deduct once only for failure to round as instructed.*

(a)

Missing values: 1.01, 0.49, -0.11

B1 Any 2 correct B1 third correct. Should be 2 dp - deduct one mark gained if any are correct but not rounded (ie would round to the correct answer). Truncated answers are wrong.

(b)

B1ft for plotting **their** values

B1ft for a smooth curve through **all** their plotted points. Ignore any graph to left of  $x = -1$  or to the right of 5 as these are outside the given domain

(c)

M1 for making the given equation match the curve equation. (May not be  $\frac{3}{4}$  on RHS.)

A1cso for  $x = 2.5$  Ignore any answers outside  $-1 \leq x \leq 5$  Must be 1 dp unless already penalised for incorrect rounding in (a). Check their answer agrees with their graph before giving A1.

(d)

M1 for making the given equation match the curve equation. (May not be  $\frac{3}{2}x$  on RHS but must be  $5 \log(x+2) - x$  on LHS)

A1 for  $5 \log(x+2) - x = \frac{3}{2}x$

M1dep for drawing the line  $y = \mathbf{their}$  rhs on their graph.

A1cso for  $x = 0.9$  Must have been obtained by drawing the correct line which must pass through the origin. Must be 1 dp unless rounding already penalised.

**NB** Because of the graph some candidates are labelling (c) as (b) and (d) as (c). Do not penalise this.

Question Number	Scheme	Marks
8.	<p>(a) Gradient of <math>l_1</math> is <math>-\frac{2}{3}</math></p> <p>(b) Gradient of <math>l_2</math> is <math>-\frac{1}{-\frac{2}{3}} = \frac{3}{2}</math>  Equation of <math>l_2</math> is <math>y - 2 = \frac{3}{2}(x - 7)</math>      <math>[2y = 3x - 17]</math></p> <p>(c) <math>2x + 3y = -6 \Rightarrow 6x + 9y = -18</math>  <math>3x - 2y = 17 \Rightarrow 6x - 4y = 34</math>  <math>13y = -52</math>  <math>y = -4</math>  <math>2x - 12 = -6 \Rightarrow x = 3</math>      <math>[Q(3, -4)]</math></p> <p>(d) Equation of <math>l_3</math> is <math>y - 2 = -\frac{2}{3}(x - 7)</math>      <math>[3y + 2x = 20]</math>  or <math>2x + 3y + k = 0</math> so at <math>P(7, 2)</math>, <math>14 + 6 + k = 0</math>      M1  <math>k = -20 \Rightarrow 2x + 3y - 20 = 0</math>      A1</p> <p>(e) at <math>R</math>, <math>y = 0</math> so <math>2x + 6 = 0 \Rightarrow x = -3</math>      B1  <math>QR^2 = (3 + 3)^2 + (-4 - 0)^2</math> and <math>PQ^2 = (7 - 3)^2 + (2 + 4)^2</math>      M1  <math>\Rightarrow QR = \sqrt{36 + 16} = \sqrt{52}</math> and <math>PQ = \sqrt{4^2 + 6^2} = \sqrt{52}</math> or <math>QR^2 = PQ^2 = 52</math>      A1</p> <p>(f) <math>PQRS</math> is a square      B1  so area = <math>PQ \times QR = \sqrt{52} \times \sqrt{52}</math>      M1  = 52      A1</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1      (15)</p>

### Notes for Question 8

(a)

B1 for  $-\frac{2}{3}$  (or - 0.6 rec. or - 0.667, seen occasionally). Must be shown explicitly (re-arranging the equation to  $y = -\frac{2}{3}x - 2$  is not sufficient).

(b)

M1 for finding the gradient of  $l_2$  as  $-\frac{1}{\text{their gradient of } l_1}$

M1 for any complete method for finding the equation of  $l_2$ . Award M0 if gradient of  $l_1$  is used. Use of  $y = mx + c$  needs a value for  $c$  to be found.

A1cso for  $y - 2 = \frac{3}{2}(x - 7)$  oe. No need to simplify so ignore any simplification shown.

(c)

M1 for attempting the solution of the pair of simultaneous equations

A1 for  $x = 3$  or  $y = -4$

A1 for the second value correct. No need to write in coordinate brackets

(d)

M1 for attempting the equation of  $l_3$  - any complete valid method.

A1 for  $y - 2 = -\frac{2}{3}(x - 7)$  oe (Ignore any simplification shown)

(e)

B1 for  $x = -3$  No working need be shown.

M1 for attempting to obtain the length of either  $PQ$  or  $QR$  or  $PQ^2$  or  $QR^2$  using **their** coordinates of  $Q$ .

A1cao for both lengths or squares of lengths correct.

(f)

B1 state that or use the fact that  $PQRS$  is a square

M1 for Area =  $PQ \times QR = \dots$  using their values

A1cso for 52

### Notes for Question 8 Continued

*Alternatives:*

1. "Determinant" method:

B1 for  $S$  is (1, 6) seen explicitly or used

M1 for using **their** coordinates for  $P$ ,  $Q$ ,  $R$  and  $S$  in the "determinant"

- must have the points in order, clockwise or anticlockwise
- must have a closed shape, ie first and last in the determinant are the same
- must use  $\frac{1}{2}$ , either now or to complete the work

Example "determinant" 
$$\frac{1}{2} \begin{vmatrix} 7 & 3 & -3 & 1 & 7 \\ 2 & -4 & 0 & 6 & 2 \end{vmatrix}$$

A1 for 52 (must be positive)

2. Drawing a square around  $PQRS$ , finding area of this square and subtracting the triangular corners.

B1 for  $S$  is (1, 6) seen explicitly or used

M1 for a complete method, ie find **all** required areas and attempt the subtractions needed

A1 for 52

Question Number	Scheme	Marks
9.	(a) (i) $(1+x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{1 \times 2}x^2 + \frac{(-1)(-2)(-3)}{1 \times 2 \times 3}x^3 \dots$ $= 1 - x + x^2 - x^3 \dots$ (ii) $(1-2x)^{-1} = 1 - (-2x) + (-2x)^2 - (-2x)^3 \dots$ $= 1 + 2x + 4x^2 + 8x^3 \dots$	M1 A1 M1 A1
	(b) $\frac{2}{1-2x} + \frac{1}{1+x} = \frac{2(1+x) + (1-2x)}{(1-2x)(1+x)}$ $= \frac{3}{(1-2x)(1+x)}$ so $A=0$ and $B=3$	M1 A1
	(c) (i) $\frac{1}{(1-2x)(1+x)}$ $= \frac{1}{3} \left( \frac{2}{1-2x} + \frac{1}{1+x} \right)$ or $(1+2x+4x^2+8x^3 \dots)(1-x+x^2-x^3 \dots)$ $= \frac{1}{3} (2(1+2x+4x^2+8x^3 \dots) + (1-x+x^2-x^3 \dots))$ $= \frac{1}{3} (3+3x+9x^2 \dots)$ $= 1+x+3x^2 \dots$ or $1-x+x^2+2x-2x^2+4x^2 \dots$	M1 M1dep A1
	(ii) valid when $ x  < \frac{1}{2}$	B1
	(d) $\int_{0.1}^{0.2} \frac{1}{(1-2x)(1+x)} dx \approx \int_{0.1}^{0.2} (1+x+3x^2) dx$ $= \left[ x + \frac{1}{2}x^2 + x^3 \right]_{0.1}^{0.2}$ $= (0.2 + 0.02 + 0.008) - (0.1 + 0.005 + 0.001)$ $= 0.122$	M1 A1 M1dep A1 (14)

### Notes for Question 9

(a)(i) M1 for attempting the binomial expansion. Must have 1 and denominators  
2! or 2 (with  $x^2$ ) and 3! or 6 (with  $x^3$ )

A1 for  $1 - x + x^2 - x^3$

(ii) M1 for replacing  $x$  with  $\pm 2x$  in the expansion obtained in (a) **OR** use the binomial expansion again - rules as above and  $(\pm 2x)^k$   $k > 0$  in at least one term.

A1 for  $1 + 2x + 4x^2 + 8x^3$

(b)

M1 for adding the two fractions to form a single fraction

A1 for  $A = 0$ ,  $B = 3$

(c)(i)

M1 for either  $\frac{1}{(1-2x)(1+x)} = (\text{product of their expansions from (a)})$

or  $\frac{1}{3} \left( \frac{2}{1-2x} + \frac{1}{1+x} \right)$  (allow if  $\frac{1}{3}$  missing, as long as it appears later)

M1dep for multiplying **their** expansions from (a) - min 5 terms, no simplification yet, or adding  $2 \times$   
**their** expansion of  $(1 - 2x)^{-1}$  to **their** expansion of  $(1 + x)^{-1}$  (allow if  $\frac{1}{3}$  missing, as long as it appears  
later)

A1 for  $1 + x + 3x^2$  or  $\frac{1}{3}(3 + 3x + 9x^2)$  Ignore higher powers.

(ii) B1 for  $|x| < \frac{1}{2}$  oe use of  $\leq$  gets B0

(d)M1 for integrating **their** expansion from (c)- minimum 2 terms

A1ft for correct integration of **their** expansion

M1dep for substituting the correct limits in their result

A1cso for 0.122 **must** be 3 dp.

**NB:** Use of calculator for (d):

If the correct results have been obtained in (c) and (d), award 4/4 for (d)

If either the expansion in (c) or the result in (d) is incorrect, award 0/4. (No part marks when insufficient working is shown.)

Question Number	Scheme	Marks
10.	<p>(a) <math>s = \sqrt{3} \sin \frac{\pi}{6} + \cos \frac{\pi}{6} = \sqrt{3} \times \frac{1}{2} + \frac{\sqrt{3}}{2}</math>  <math>= \sqrt{3}</math></p> <p>(b) At <math>O</math>, <math>s = 0</math> so <math>\sqrt{3} \sin \frac{1}{2}t + \cos \frac{1}{2}t = 0</math>  <math>\Rightarrow \tan \frac{1}{2}t = -\frac{1}{\sqrt{3}}</math>  <math>\Rightarrow \frac{1}{2}t = \frac{5\pi}{6}</math>  <math>\Rightarrow t = \frac{5\pi}{3}</math></p> <p>(c) <math>v = \frac{ds}{dt} = \frac{\sqrt{3}}{2} \cos \frac{t}{2} - \frac{1}{2} \sin \frac{t}{2}</math></p> <p>(d) <math>\cos\left(\frac{\pi}{6} + \frac{t}{2}\right) = \cos \frac{\pi}{6} \cos \frac{t}{2} - \sin \frac{\pi}{6} \sin \frac{t}{2}</math>  <math>= \frac{\sqrt{3}}{2} \cos \frac{t}{2} - \frac{1}{2} \sin \frac{t}{2} = v</math></p> <p>(e) <math>\cos\left(\frac{\pi}{6} + \frac{t}{2}\right) = \frac{1}{2}</math>  <math>\Rightarrow \frac{\pi}{6} + \frac{t}{2} = \frac{\pi}{3}, \frac{5\pi}{3}, \dots</math>  <math>\Rightarrow \frac{t}{2} = \dots, \frac{\pi}{6}, \frac{9\pi}{6}, \dots</math> or <math>\frac{\pi}{3} + t = \dots, \frac{2\pi}{3}, \frac{10\pi}{3}, \dots</math>  <math>\Rightarrow t = \dots, \frac{\pi}{3}, \frac{9\pi}{3}, \dots</math></p> <p>(i) <math>t = \frac{\pi}{3}</math></p> <p>(ii) <math>t = 3\pi</math></p>	<p>M1 A1</p> <p>M1 A1 M1dep A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 M1dep A1 A1 (14)</p>



### Notes for Question 10

(a) M1 for substituting  $\frac{\pi}{3}$  into the given expression for  $s$  and proceeding to a value or numerical expression (not necessarily correct) for  $s$ .

A1cao for  $s = \sqrt{3}$  No decimal values must be seen in the working. This can be done on a calculator - if  $\sqrt{3}$  is the only numerical value seen award M1A1, but if a decimal approximation is seen first and no substitution shown award M0A0.

(b) M1 for setting  $s = 0$  and rearranging to  $k \tan \frac{1}{2}t = \dots$  where  $k$  is a number

A1 for  $\tan \frac{1}{2}t = -\frac{1}{\sqrt{3}}$  oe

M1dep for obtaining a positive value for  $\frac{1}{2}t$  or  $t$  (need not be exact and may be in degrees).

Should be correct for their  $\tan \frac{1}{2}t$ . Dependent on the first M mark in (b)

A1cao for identifying the required value as  $\frac{5\pi}{3}$  (must be exact and in radians). Ignore any answers greater than  $\frac{5\pi}{3}$ . Award A0 if previous mark has been given for a decimal approx. Allow if the initial solution was in degrees and now changed to radians.

*Alternative for (b):*

$\tan \frac{\pi}{3} \sin \frac{1}{2}t + \cos \frac{1}{2}t = 0$	
$\sin \frac{\pi}{3} \sin \frac{1}{2}t + \cos \frac{\pi}{3} \cos \frac{1}{2}t = 0$	M1
$\cos \left( \frac{\pi}{3} - \frac{1}{2}t \right) = 0$	A1
$\frac{\pi}{3} - \frac{1}{2}t = -\frac{1}{2}\pi, \frac{1}{2}\pi$	M1 (either, in degrees or radians)
$t = \frac{5\pi}{3}$	A1cao Ignore extras as above

### Notes for Question 10 Continued

(c)

M1 for attempting the differentiation. cos should become sine **and** sine become cos. Allow if + between terms or  $\frac{1}{2}$  missing but not if either term is multiplied by 2 (implies integration)

A1 for  $v = \frac{\sqrt{3}}{2} \cos \frac{1}{2}t - \frac{1}{2} \sin \frac{1}{2}t$

(d)

M1 for expanding  $\cos\left(\frac{\pi}{6} + \frac{1}{2}t\right)$  with the given formula Must show  $\cos \frac{\pi}{6} \cos \frac{1}{2}t$  etc

A1 for using values for  $\cos \frac{\pi}{6}$  and  $\sin \frac{\pi}{6}$  to obtain  $v$ . \*

If worked from  $v = \frac{\sqrt{3}}{2} \cos \frac{1}{2}t - \frac{1}{2} \sin \frac{1}{2}t$ , award M1 for changing  $\frac{\sqrt{3}}{2}, \frac{1}{2}$  to trig functions **and** using the addition formula and A1 if everything is correct.

(e)

M1 for obtaining a value in radians for  $\frac{\pi}{6} + \frac{1}{2}t$  - need not be exact

M1dep for obtaining a value for  $t$  - need not be exact

(i) A1cao for  $t = \frac{\pi}{3}$       (ii) A1cao for  $t = 3\pi$

Ignore labels (i) and (ii). If more values given ignore if outside the required ranges. Deduct one or both A1 marks for each extra solution seen within the range.

If starting from their result in (c), they need to reach  $\cos\left(\frac{\pi}{6} + \frac{t}{2}\right) = \frac{1}{2}$  in order to make further progress, so award marks as for the main method.

**NB:** marks for (d) can only be given in (d).



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